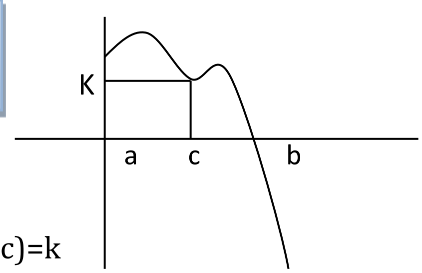
Important Theorems:

English Translation: Each y value has at least one x value



Intermediate Value Theorem

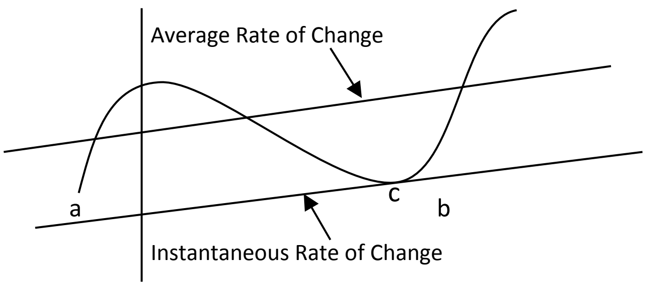
Hypothesis- f(x) is continuous on [a,b]
k is any number between f(a) and f(b)

Conclusion- There is at least one number c in [a,b] such that f(c)=k

Mean Value Theorem

Hypothesis- f(x) is continuous on [a,b]
f(x) is differentiable on (a,b)

Conclusion- There exists a number c, with a<c<b, such that $f'(c) = \frac{f(b)-f(a)}{b-a}$



In other words, the instantaneous rate of change at point c equals the average rate of change, because the two lines are parallel.

Extreme Value Theorem

Hypothesis- f(x) is continuous on [a,b]

Conclusion- f(x) has a global maximum and global minimum on [a,b]

Note: Evaluate the function at the critical points and at the endpoints, a and b

Racetrack Principle

Hypothesis- g(x) and h(x) are continuous on [a,b]
g(x) and h(x) are differentiable on (a,b)
$g'(x) \leq h'(x)$ on (a,b)

Conclusion- If g(a) = h(a), then $g(x) \leq h(x)$ for [a,b]
If g(b) = h(b), then $g(x) \geq h(x)$ for [a,b]

We can think of g(x) and h(x) as the positions of two racehorses at time x, with horse h always moving faster than horse g. If they start together, horse h is ahead during the whole race. If they finish together, horse g was ahead during the whole race

Derivatives Fact Sheet:

Function	***Derivative***
sin(x)	cos(x)
cos(x)	-sin(x)
tan(x)	$\sec^2(x)$
sec(x)	sec(x)·tan(x)
csc(x)	-csc(x)·cot(x)
cot(x)	$-\csc^2(x)$
$\sin^{-1}(x)$ Domain: [-1,1] Range: $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}(x)$ Domain: [-1,1] Range: $[0,\boldsymbol{\pi}]$	$\frac{1}{-\sqrt{1-x^2}}$
$\tan^{-1}(x)$ Domain: $x\in \boldsymbol{R}$ Range: $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$	$\frac{1}{1+x^2}$
$\sec^{-1}(x)$ Domain: $(-\infty,-\mathbf{1}]\cup[\mathbf{1},\infty)$ Range: $\left[\mathbf{0},\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\boldsymbol{\pi}\right]$	$\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}}$
a^x	$\ln(a)\cdot a^x$
ln(x)	$\frac{1}{x}$
$\log_b(x)$	$\frac{1}{\ln(b)\cdot x}$
$f^{-1}(x)$	$\frac{1}{f'(f^{-1}(x))}$

$Product\ Rule: (fg)' = f'g + fg'$ | $Quotient\ Rule: \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$Chain\ Rule: \frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$ | $\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}$

Note: f(x) and g(x) must be differentiable

f	*f'*	*f''*
increasing	positive	
decreasing	negative	
concave up	increasing	positive
concave down	decreasing	negative
"flat spot" (stationary pt)	0	
Inflection point	local minimum	Negative to positive

	local maximum	Positive to negative
local maximum	positive to negative	(If f' exists, f'=0) negative
local minimum	negative to positive	(If f' exists, f'=0) positive

Note: The local min or max could be a point, such as in |x|

Using Derivatives:

Limit Definition

$$f'(x) = Rate\ of\ change\ of\ f\ at\ x = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

- A function is differentiable if the limit above exists
  - A differentiable function is continuous
- The slope of a linear function is its derivative because that is the rate at which it's changing
- Δ notation

Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

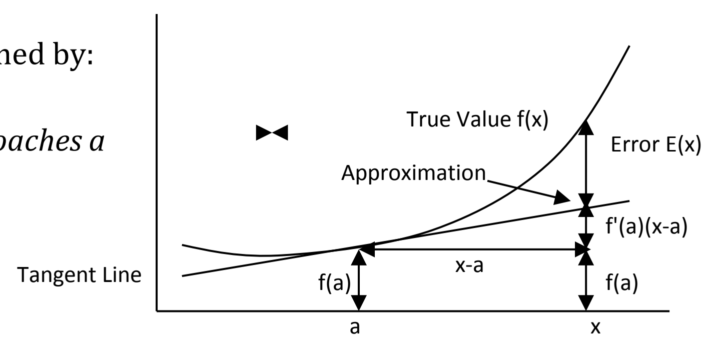
Velocity: v(t)= $\frac{dy}{dx}$ Acceleration: a(t) = $\frac{d^2y}{dt^2}$

Local Linearization

If f is differentiable at a, then for values near a, the tangent line approximation to f(x) is
$f(x) \approx f(a) + f'(a)(x-a)$

The **error** in the approximation is defined by:
$E(x) = f(x) - f(a) - f'(a)(x-a)$

Note: The error approaches 0 as x approaches a



*Key Term: **critical point-** a point p in the domain of f where f'(p) = 0 or f'(p) is undefined. The point (p, f(p)) is also called a critical point, while f(p) is called a critical value*

1st Derivative Test for Local Maxima and Minima	2nd Derivative Test for Local Maxima and Minima
Suppose p is a critical point of continuous function f • If f' changes from negative to positive at p, then f has a local min at p • If f' changes from positive to negative at	If f'(p)=0, and • f''(p) > 0, then f has a local min at p • f''(p) < 0, then f has a local max at p • f''(p) = 0, then the test tells us nothing

p, then f has a local max at p	

Suppose a function f has a continuous derivative. If f'' changes sign at p, then f has an inflection point at p, and f' has a local minimum or a local maximum at p.

Note: Use a closed interval for increasing/decreasing and an open interval for differentiability

Limits

Properties

Assuming limits on the right hand side exist

1. If b is a constant, then $\lim_{x \to c} bf(x) = b(\lim_{x \to c} f(x))$
2. $\lim_{x \to c}(f(x) + g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
3. $\lim_{x \to c}(f(x)g(x)) = (\lim_{x \to c} f(x))(\lim_{x \to c} g(x))$
4. $\lim_{x \to c} \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}, provided\ \lim_{x \to c} g(x) \neq 0$

Definition of Continuity

A function f is continuous at a point x=a if

1. f(a) exists,
2. $\lim_{x \to c} f(x)$ exists, and
3. $\lim_{x \to c} f(x) = f(a)$

In other words: the limit as x approaches c from the right side must equal the limit as x approaches c from the left side, which must equal f(a)

- Sums, products, quotients, and composite functions of continuous functions on an interval are continuous on that interval.
- Continuity is important because it allows us to make conclusions, such as that the function passes through zero when it goes from positive to negative

Vertical Asymptotes- set denominator equal to zero and solve for x
Horizontal Asymptotes-Divide the coefficient of the highest degree by the lower one of the same degree (if it's the highest in the denominator)
Involving Infinity- multiply numerator and denominator by $\frac{1}{x^{largest\ power}}$ to determine limit

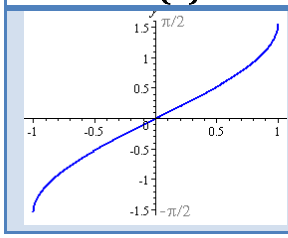
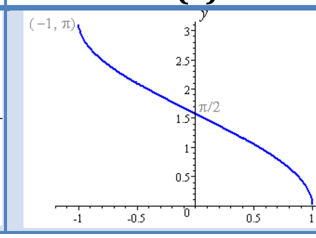
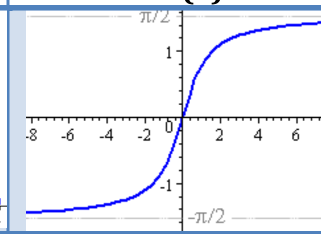
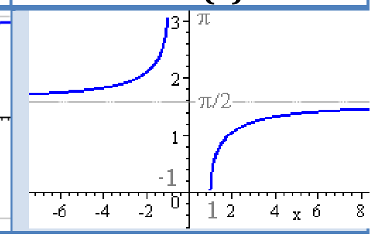
Special Trigonometric Limits

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1 \; and \; \lim_{\theta \to 0} \frac{1 - \cos(\theta)}{\theta} = 0$$

Formal Definition- $\lim_{x \to a} f(x) = L$ $if, given\ any\ \varepsilon > 0, we\ can\ find\ \delta > 0\ such\ that$ $if\ 0 < |x - a| < \delta, then\ |f(x) - L| < \varepsilon$

Functions

Graphs of Inverse Trig Functions

sin⁻¹(x)	cos⁻¹(x)	tan⁻¹(x)	sec⁻¹(x)
			

Euler's Method

$$y_{new} = y_{old} + \Delta y = y_{old} + m \cdot \Delta x$$

Absolute Value

Example:

$$|x - p| < q$$

This inequality contains all the numbers that are less than q units away from p

$$|x - p| < |x - q|$$

The distance from x to p is less than the distance from x to q